

A Nearly Model-Independent Characterization of Dark Energy Properties as a Function of Redshift

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Understanding the acceleration of the universe and its cause is one of the key problems in physics and cosmology today, and is best studied using a variety of mutually complementary approaches. Daly and Djorgovski (2003, 2004) proposed a model independent approach to determine the expansion and acceleration history of the universe and a number of important physical parameters of the dark energy as functions of redshift directly from the data. Here, we apply the method to explicitly determine the first and second derivatives of the coordinate distance with respect to redshift, y' and y'' , and combine them to solve for the kinetic and potential energy density of the dark energy as functions of redshift, $K(z)$ and $V(z)$.

A data set of 228 supernova and 20 radio galaxy measurements with redshifts from zero to 1.79 is used for this study. Values of y' and y'' are combined to study the dimensionless acceleration rate of the universe as a function of redshift, $q(z)$. The only assumptions underlying our determination of $q(z)$ are that the universe is described by a Robertson-Walker (RW) metric and is spatially flat. We find that the universe is accelerating today, and was decelerating in the recent past. The transition from acceleration to deceleration occurs at a redshift of about $z_T = 0.42 \pm 0.08$. Values of y' and y'' are combined to determine $K(z)$ and $V(z)$. These are shown to be consistent with the values expected in a standard Lambda Cold Dark Matter (LCDM) model.

The acceleration of the universe at the present epoch has been studied in the contexts of specific models using coordinate distances to type Ia supernovae [1–7], and FR II radio galaxies [8–11], in addition to other techniques. These studies indicate that the universe is expanding at an accelerating rate at the present epoch. Generally, these studies are done in the context of a specific cosmological model, such as an open universe with non-relativistic matter, a cosmological constant, and space curvature (e.g. [1,2,8,9]), a spatially flat universe with non-relativistic matter and dark energy that has an energy density that can evolve with redshift but which maintains a constant equation of state (e.g. [2–7,10]), or a spatially flat universe with non-relativistic mat-

ter and an evolving scalar field (e.g. [12,11]). In each of these studies it is implicitly assumed that the universe is described by a RW metric and that General Relativity is the correct theory of gravity; in addition, a particular functional form for the redshift evolution of the energy density of some new component is assumed. The data are then used to constrain the parameters that describe the assumed functional form for the redshift evolution of whatever was being considered as the driver of the acceleration of the universe.

A complementary approach was suggested by [13,14] who showed that the recent expansion and acceleration history of the universe, and some properties of the driver of the acceleration, can be determined directly from the data after specifying a minimal number of assumptions. Assuming only that the universe is described by a RW metric and is spatially flat, the data can be used to solve for the dimensionless expansion and acceleration rates of the universe as functions of red-

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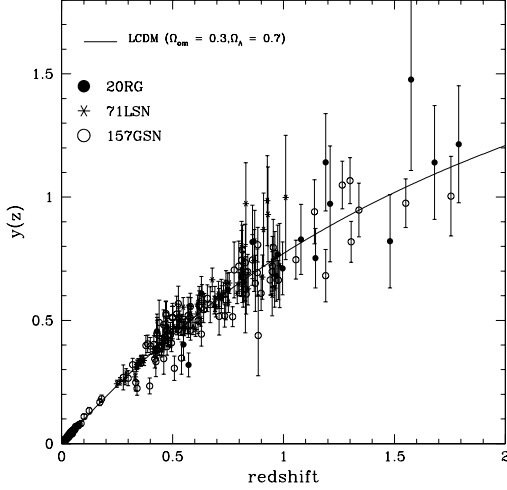


Figure 1. Dimensionless coordinate distance $y(z)$ to 71 Legacy and 157 Gold supernovae, and 20 radio galaxies; $y(z) \equiv H_0(a_0 r)$, H_0 is Hubble's constant, and $(a_0 r)$ is the coordinate distance to a source at redshift z . It is convenient to work with $y(z)$ because it is independent of H_0 (i.e. $(a_0 r) \propto H_0^{-1}$, so $H_0(a_0 r)$ is independent of H_0).

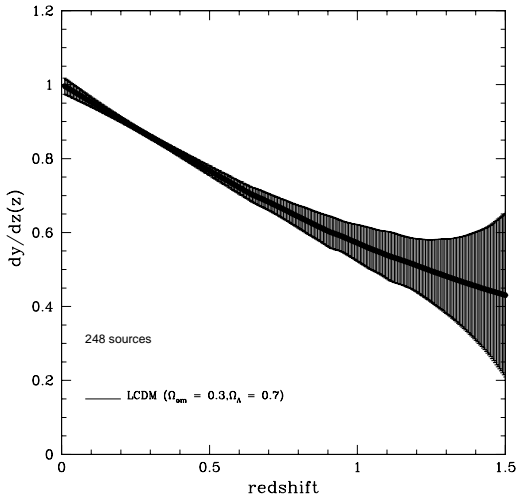


Figure 2. Results obtained with the mock data set of 248 sources described in the text. The results are in an excellent agreement with the input cosmology, with no apparent bias.

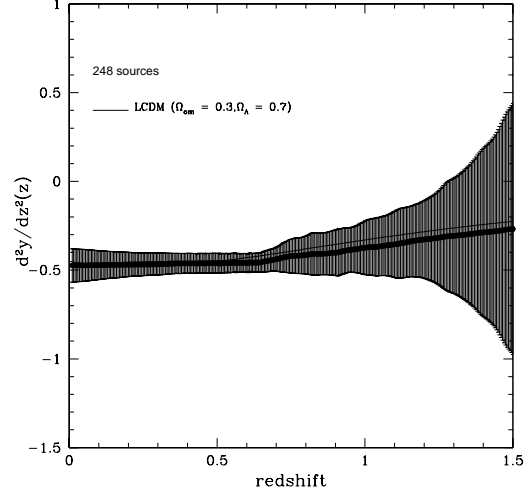


Figure 3. As in Fig. 2 for y'' . The correct assumed cosmology is recovered with a negligible bias.

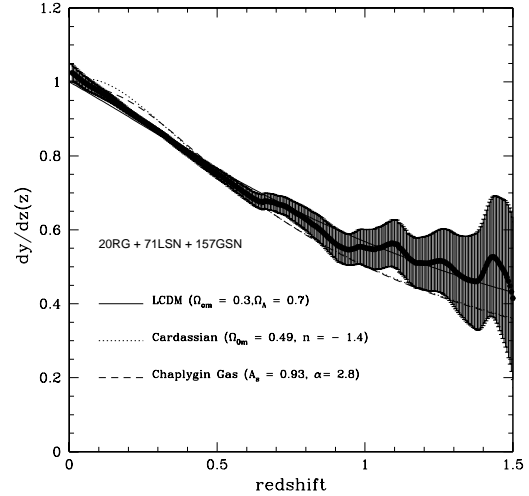


Figure 4. The first derivative of the coordinate distance with respect to redshift for the actual data set of 248 sources. The zero redshift value we measure is $y'_0 = 1.025 \pm 0.022$; the predicted value in all models is 1.000. The values for the standard LCDM model with $\Omega_\Lambda = 0.7$ and $\Omega_{0m} = 0.3$ are shown as the solid line in this and all subsequent plots. Best fit Cardassian (dotted line) and Chaplygin Gas (dashed line) models are also shown, and are described in the text.

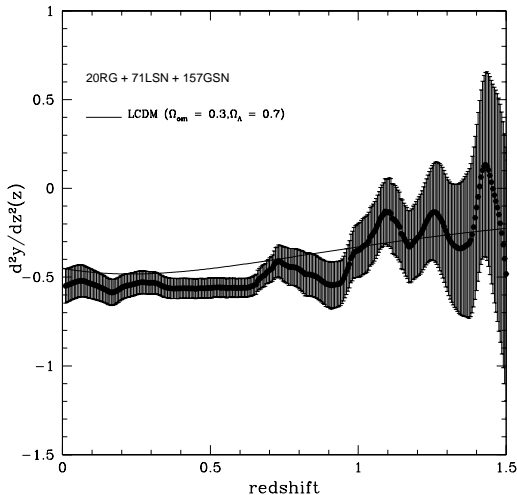


Figure 5. The second derivative of the coordinate distance with respect to redshift for the actual data set of 248 sources. The measured zero redshift value is $y''_0 = -0.55 \pm 0.10$; the value predicted in the LCDM model shown is -0.45 .

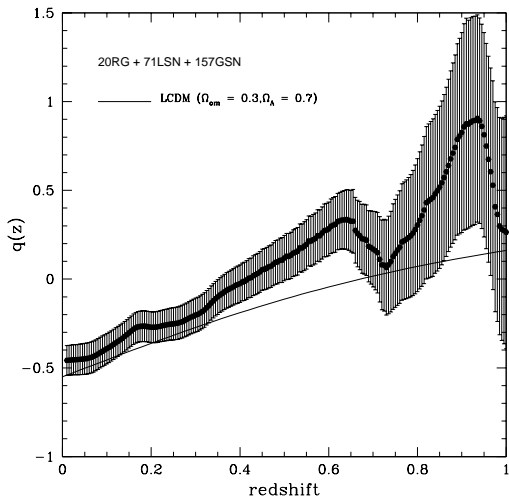


Figure 6. The deceleration parameter $q(z)$, where $q(z) = -[1 + y''(1+z)/y']$ [13]. The zero redshift value is $q_0 = -0.46 \pm 0.08$. The predicted value in the LCDM model shown is -0.55 . Our fits are systematically higher than the LCDM model shown by about 1σ .

shift, $E(z)$ and $q(z)$, respectively. This can be done without specifying a theory of gravity, or anything else. The function $q(z)$ thus obtained is a direct measure of the acceleration/deceleration of the universe at different epochs. The key ingredients that go into the determination of $E(z)$ and $q(z)$ are the first and second derivatives of the coordinate distance with respect to redshift, dy/dz (or y') and d^2y/dz^2 (or y''), which are obtained from the coordinate distances to supernovae and radio galaxies at known redshift, as described by [13,14]. Thus, rather than assuming a functional form for the redshift evolution of the “dark energy” and constraining the model parameters, it is possible to solve for quantities such as $q(z)$ directly.

This direct approach indicates that the universe is accelerating today, and was decelerating in the recent past. The data used for the results shown here include the sample of 157 “Gold” supernovae [6], the sample of 71 supernovae from the Supernova Legacy Survey [7], and the 20 radio galaxies of [9], as described in detail [15]. The total sample of 248 sources is shown in Fig. 1; there are no systematic differences seen among the three groups of measurements in the redshift ranges of their overlaps.

The first and second derivatives of the coordinate distance with respect to redshift are obtained using the numerical differentiation method described by [13,14]. To test whether the method introduces a bias in the results, a mock data set of 248 sources with the same redshift distribution and fractional uncertainty per point as the actual data was constructed assuming a LCDM model with $\Omega_{0m} = 0.3$ and $\Omega_{\Lambda} = 0.7$, and analyzed. The results are shown in Figs. [2] and [3]. We see that no bias has been introduced by the numerical differentiation technique.

Values of y' and y'' are shown on Figs. [4] and [5]. The ringing seen in these figures is most likely due to sparse sampling. In these plots, and in the ones that follow, we do not consider these fluctuations at higher redshifts to be statistically significant, as they are commensurate with our derived $1\text{-}\sigma$ error bars. The results are consistent with the LCDM model. The LCDM model is based on General Relativity with non-relativistic mat-

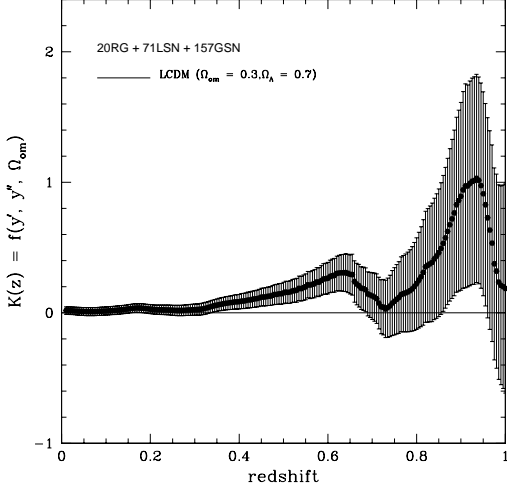


Figure 7. The kinetic energy of the dark energy $K(z)$ in units of the critical density today. The zero redshift value is $K_0 = 0.02 \pm 0.03$; the expected value for the LCDM model shown 0.

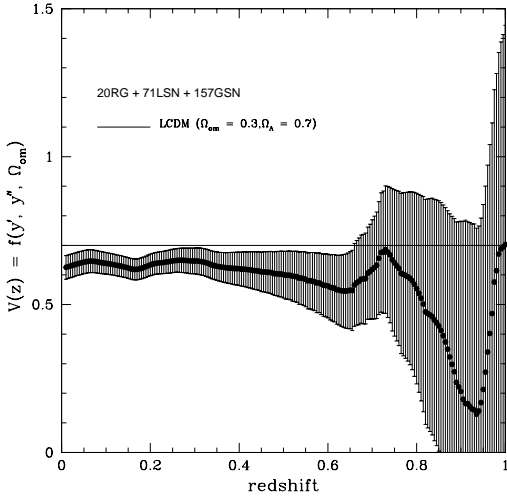


Figure 8. The potential energy of the dark energy $V(z)$ in units of the critical density today. The zero redshift value is $V_0 = 0.62 \pm 0.05$; the expected value for the LCDM model shown is 0.7.

ter Ω_{0m} and a cosmological constant. This provides an excellent description of the data. Curves showing predictions in two modified gravity models in a spatially flat universe are also shown on Fig. [4]. These curves shown are obtained using the best fit model parameters obtained by [16] for the Cardassian model of [17] and the generalized Chaplygin gas model of [18] based on the model of [19]; this is consistent with the results obtained by [20]. Clearly, the LCDM model provides a better description of the data than do either of the modified gravity models. Thus, this large-scale test of General Relativity shows that GR provides an excellent description of the data on these very large length scales of about 10 billion light years.

The deceleration parameter $q(z)$ is shown on Fig.[6]. These results allow a determination of the redshift at which the universe transitions from an accelerating phase to a decelerating phase; we find this redshift to be $z_T = 0.42 \pm 0.08$, consistent with the values quoted by [6] and [13,14]. The upper bound on this transition redshift is uncertain because of the fluctuations in $q(z)$ which are due to sparse sampling at high redshift.

It is well known that $K = 0.5(\rho + P)$ and $V = 0.5(\rho - P)$, where ρ and P are the energy density and pressure of the dark energy. In [14] we show that both ρ and P may be written in terms of the first and second derivatives of the coordinate distance. Combining these, we find that $(K/\rho_{oc}) = -(1+z)(y'')(y')^{-3}/3 - 0.5\Omega_{0m}(1+z)^3$, and $(V/\rho_{oc}) = (y')^{-2}[1 + (1+z)y''(y')^{-1}/3] - 0.5\Omega_{0m}(1+z)^3$, where ρ_{oc} is the critical density at the current epoch. These are shown in Figs. [7] and [8]. In obtaining K and V , the assumptions made to obtain P and ρ apply: the universe is spatially flat; the kinematics of the universe are accurately described by general relativity; and two components, the dark energy and non-relativistic matter (with $\Omega_{0m} = 0.3$), are sufficient to account for the kinematics of the universe out to redshift of about 2 (see the discussion in [14]). Functional forms for $P(z)$ and $\rho(z)$ for the dark energy are *not* assumed, nor is any assumption made regarding the equation of state of the dark energy. The work presented here on the potential energy, $V(z)$, is complementary to the work of [21–24].

Thus, our (nearly) model-independent method provides results which are consistent with those from the more traditional approaches, in a largely complementary fashion; at the very least, it is a new way of looking at the data. As the quality and size of relevant data sets increase, we can expect even more useful constraints to emerge from this approach.

REFERENCES

1. A. G. Riess, et al., *AJ*, **116**, 1009 (1998).
2. S. Perlmutter et al., *ApJ*, **517**, 565 (1999).
3. J. T. Tonry et al., *ApJ*, **594**, 1 (2004).
4. R. A. Knop et al., *ApJ*, **598**, 102 (2003).
5. B. J. Barris, et al., *ApJ*, **602**, 571 (2004).
6. R. G. Riess, et al., *ApJ*, **607**, 665 (2004).
7. P. Astier, et. al, *Astron. & Astrophys.*, **447**, 31 (2006).
8. R. A. Daly, E. J. Guerra, and L. Wan, in Proc. 33d Rencontre de Moriond *Fundamental Parameters in Cosmology*, eds. J. Tran Thanh Van, Y. Giraud-Heraud, F. Bouchet, T. Damour, and Y. Mellier (Paris: Editions Frontieres), 323 (1998); astro-ph/9803265
9. E. J. Guerra, R. A. Daly, and L. Wan, *ApJ*, **544**, 659 (2000).
10. R. A. Daly, and E. J. Guerra, *AJ*, **124**, 1831 (2002).
11. S. Podariu, R. A. Daly, M. P. Mory, and B. Ratra, *ApJ*, **584**, 577 (2003).
12. S. Podariu, and B. Ratra, *ApJ*, **532**, 109 (2000).
13. R. A. Daly and S. G. Djorgovski, *ApJ*, **597**, 9 (2003).
14. R. A. Daly and S. G. Djorgovski, *ApJ*, **612**, 652 (2004).
15. R. A. Daly and S. G. Djorgovski, astro-ph/0512578 (2005).
16. M. C. Bento, O. Bertolami, N. M. C. Santos, and A. A. Sen, astro-ph/0512076 (2005).
17. K. Freese and M. Lewis, *Phys. Lett. B.*, **540**, 1 (2002).
18. M. C. Bento, O. Bertolami, and A. A. Sen, *Phys. Rev. D.*, **66**, 043507 (2002).
19. A. Y. Kamenshchik, U. Moschella, and V. Pasquier, *Phys. Lett. B.*, **511**, 265 (2001).
20. R. Lazkoz, S. Nesseris, and L. Perivolaropoulos, *JCAP*, **11**, 10 (2005).
21. T. Saini, S. Raychaudhury, V. Sahni, and A. A. Starobinsky, *Phys. Rev. Lett.*, **85**, 1162 (2000).
22. V. F. Cardone, A. Troisi, and S. Cappozziello, *Phys. Rev. D* **69**, 3517 (2004).
23. J. Simon, L. Verde, and R. Jimenez, *Phys. Rev. D* **71**, 123001 (2005).
24. M. Sahlen, A. R. Liddle, and D. Parkinson, *Phys. Rev. D* **72**, 083511 (2005).