Ferroelectrics, 1999, Vol. 222, pp. 87-93
Reprints available directly from the publisher
Photocopying permitted by license only

Classification of Domain Pairs and Tensor Distinction

D.B. LITVIN[a] and V. JANOVEC[b]

[a]*Department of Physics, The Pennsylvania State University, Penn State Berks Campus, P.O. Box 7009, Reading, PA 19610–6009, U.S.A.; and* [b]*Technical University of Liberec, Halková 6, 461 17 Liberec 1, Czech Republic*

The symmetries of domain states of a domain pair (S_1,S_2) can be determined from the symmetry group $\mathbf{F}_1$ of the first domain state and a twinning operation g_{12} which transforms this domain state into the second. A classification is introduced which classifies all possible domain pairs according to a classification of the pairs $\mathbf{F}_1$ and g_{12}. One obtains 139 classes of which 43 correspond to non-ferroelastic domain pairs and 96 to ferroelastic domain pairs. The tensor distinction of domain states of a domain pair is determined by the group $\mathbf{F}_1$ and g_{12} and all domain pairs belonging to the same class have the same tensor distinction. The 139 classes of pairs $\mathbf{F}_1$ and g_{12} are tabulated along with their corresponding twinning group $\mathbf{K}_{12} = < \mathbf{F}_1, g_{12} >$, the group generated by $\mathbf{F}_1$and g_{12}. The tensor distinction of domain states of domain pairs of each class is given for a variety of material property tensors.

Keywords: domains; tensors; symmetry

INTRODUCTION

Consider a ferroic phase transition of a crystalline structure from a phase of high symmetry **G** to a phase of low symmetry **F**. In the low symmetry phase arise domains, volumes of homogeneous crystalline structure differing only enantimorphically and/or in their orientation in space. We shall refer to the bulk structures of these domains in a polydomain sample as *domain states* and denote single domain states by $S_1, S_2, ..., S_n$ where n is the index of **F** in **G**. Domain states of a domain pair can exhibit different physical properties. We shall refer to this as *tensor distinction*, i.e. the distinction of domain states of domain pairs by macroscopic tensorial physical property tensors. As these tensors depend only on the point group of the space group of the domain state, we shall interpret the symbols **G** and **F** to be the point groups of the high and low symmetry phases, respectively. We do not consider the disorientations of domains in ferroelastic samples, i.e. rotations of domains which arise as a result of the requirement that neighboring domains in polydomain samples must meet along a coherent boundary (see e.g. Janovec, Litvin, and Richterová[1]). Consequently, all domain arising in a transition from **G** to **F** are related by elements of **G**.

Aizu[2] considered *global* tensor distinction, i.e. if a specific property tensor can distinguish among all, some or none of the domains which arise in a transition from **G** to **F**. Using a classification of transitions **G** to **F**, Litvin[3] developed a method to determine such global tensor distinction and the global tensor distinction was given for all classes of transitions and property tensors of rank $n \leq 4$ (Litvin[4]).

We consider here the general case of the tensor distinction of domain pairs (S_i,S_j), two domains simultaneously observed by a single apparatus. The special case of non-ferroelastic phases have been considered elsewhere (Janovec, Richterová, and Litvin[5,6], Litvin and Janovec[7]).

TENSORIAL CLASSIFICATION OF DOMAIN PAIRS

In a phase transition from **G** to **F**, we write the coset expansion of the group **G** with respect to **F**: $\mathbf{G} = \mathbf{F} + g_2\mathbf{F} + g_3\mathbf{F} + ... + g_n\mathbf{F}$. Defining the domain state S_1 as the domain state invariant under **F** the remaining domain states are related to S_1 by the coset representatives g_i, i=2,3...,n: $S_i=g_iS_1$. Any domain state S_i is related to any other domain state S_j by an element of **G**, i.e. $S_j=g_{ij}S_i$ where $g_{ij}=g_jg_i^{-1}$. The elements g_{ij} , like the coset representatives, are not unique, since any coset representative g_i can be replaced by any element of the coset $g_i\mathbf{F}$. The domain state S_i , i=2,3,...,n is invariant under the group $F_i=g_i\mathbf{F}g_i^{-1}$. Let $\boldsymbol{T}$ denote a tensor type, and T_i, i=1,2,...,n the sets of tensor components of the tensor type $\boldsymbol{T}$ in the domain states S_i, i=1,2,...,n. The two sets of tensor components T_i and T_j of the two domain states in a domain pair (S_i,S_j) can be determined as follows: T_i is the set of tensor components of the tensor type $\boldsymbol{T}$ invariant under the group $\mathbf{F}_i$, the group under which S_i is invariant. T_j can be determined from T_i , $T_j=g_{ij}T_i$ where g_{ij} is an element of **G** which transforms the domain state S_i into the domain state S_j. ($T_j=g_{ij}T_i$ only represents the equation which relates the components of the tensors. The actual equation depends on the transformational properties of the tensor type ***T*** and its rank (Nye[8]). Consequently, in a ferroic phase transition from **G** to **F**, the tensor distinction of a domain pair (S_i,S_j) is determined by the point group $\mathbf{F}_i$ and an element g_{ij} of **G**.

We classify all possible domain pairs (S_i,S_j) into classes where all domain pairs in a single class are distinguished by components of tensors of the same set of tensor types. Two domain pairs, (S_i,S_j) whose tensor distinction is determined by the point group $\mathbf{F}_i$ and element g_{ij}, and $(S_{i'},S_{j'})$ whose tensor distinction is determined by $\mathbf{F}_{i'}$ and $g_{i'j'}$ are said to belong to the same class of domain pairs if there exists an element r of the full rotation group **R** such that

$$r\mathbf{F}_i r^{-1} = \mathbf{F}_{i'} \qquad \text{and} \qquad r g_{ij} r^{-1} = g_{i'j'} f_{i'} \qquad (1)$$

where $f_{i'}$ is an element of $\mathbf{F}_{i'}$. The appearance of the element $f_{i'}$ is due to the non-uniqueness of the choice of $g_{i'j'}$. This second condition can be replaced with a condition on cosets:

$$r g_{ij}\mathbf{F}_i r^{-1} = g_{i'j'}\mathbf{F}_{i'} \qquad (2)$$

This classification is simultaneously a classification of all pairs F_i and g_{ij}. Two domain pairs, which have corresponding F_i and g_{ij} belonging to the same class of pairs F_i and g_{ij}, belong to the same class of domain pairs. It can be shown that if two domain pairs belong to the same class, then if tensors of a given tensor type can (can not) distinguish between the domains of the first domain pair, then they can (can not) distinguish between the domains of the second domain pair. We shall call this classification the *tensorial classification* of domain pairs.

This classification of domain pairs is similar to the *crystallographic double coset classification* of domain pairs (Janovec[9]; Litvin & Wike[10]): For domain pairs arising in a transition from G to F, two domain pairs (S_i,S_j) and $(S_{i'},S_{j'})$ are said to belong to the same double coset classification of domain pairs if there is an element g of G such that $(gS_i,gS_j) = (S_{i'},S_{j'})$. Consequently, there exists an element r of G which satisfies equations (1) and (2), and therefore two domain pairs in the same double coset class are also in the same tensorial class of domain pairs. However, two domain pairs belonging to *different* double coset classes may belong to the *same* tensorial class. (On a different classification of domain pairs, see Fuksa and Janovec[11].)

The tensorial classification of domain pairs, denoted now by (S_1,S_2), classifies all domain pairs according to their corresponding $\mathbf{F}_1$ and g_{12}. There are 139 classes. In Table I we list the $\mathbf{F}_1$ and g_{12} of one domain pair from each of the 139 classes. In the first column we give a sequential numbering. An asterisk following the sequential numbering denotes the 43 classes of non-ferroelastic domain pairs, domain pairs with the same (zero) spontaneous deformation (This number of classes differs from the 48 classes of non-ferroelastic domain pairs given in Janovec, Richterová, and Litvin[6] because of the tensorial classification scheme used here.) The remaining 96 classes are of ferroelastic domain pairs. $\mathbf{F}_1$ and g_{12} are given in the next two columns. In the fourth column we give the twinning group $\mathbf{K}_{12} = \langle \mathbf{F}_1, g_{12} \rangle$, the group generated by $\mathbf{F}_1$ and g_{12}. (See Janovec, Litvin, and Fuksa[12]) for use of the twinning group in the tensor distinction of domain pairs.)

TENSOR DISTINCTION

We give the tensor distinction of domains in domain pairs for all 139 classes of domain pairs for seven tensor types:(1) ϵ enantiomorphism (2) V spontaneous polarization (3) $\epsilon[V^2]$ optical activity (4) $V[V^2]$ piezoelectricity, electrooptics (5) $\epsilon V[V^2]$ electrogyration (6) $[[V^2]^2]$ linear elasticity (7) $[V^2]^2$ piezoptics, electrostriction. In the fifth column of Table 1, for each class, is given a string of seven letters. Each letter denotes the tensor distinction of the domains of domain pairs in that class for the seven tensor types given in the above table. The sequential order of the letters corresponds to the sequential numbers of the tensor types. "Y" denotes that a physical property represented by a tensor of that tensor type can distinguish between the domains of domain pairs of that class. "N" denotes that the tensor can not distinguish between the domains. "Z" denotes that the tensor is identically zero in both domains.

Acknowledgments

D.B.L. acknowledges the support of the National Science Foundation through grant DMR-9722799 and V.J. the support of the Grant Agency of the Czech Republic through grant 202/96/0722.

Table 1

	F_1	g_{12}	K_{12}	Tensor Distinction
1)*	1	$\bar{1}$	$\bar{1}$	Y,Y,Y,Y,N,N,N
2)	1	2_z	2_z	N,Y,Y,Y,Y,Y,Y
3)	1	m_z	m_z	Y,Y,Y,Y,Y,Y,Y
4)	1	4_z	4_z	N,Y,Y,Y,Y,Y,Y
5)	1	$\bar{4}_z$	$\bar{4}$	Y,Y,Y,Y,Y,Y,Y
6)	1	3_z	3_z	N,Y,Y,Y,Y,Y,Y
7)	1	$\bar{3}_z$	$\bar{3}_z$	Y,Y,Y,Y,Y,Y,Y
8)	1	6_z	6_z	N,Y,Y,Y,Y,Y,Y
9)	1	$\bar{6}_z$	$\bar{6}_z$	Y,Y,Y,Y,Y,Y,Y
10)	$\bar{1}$	2_z	$2_z/m_z$	Z,Z,Z,Z,Y,Y,Y
11)	$\bar{1}$	4_z	$4_z/m_z$	Z,Z,Z,Z,Y,Y,Y
12)	$\bar{1}$	3_z	$\bar{3}_z$	Z,Z,Z,Z,Y,Y,Y
13)	$\bar{1}$	6_z	$6_z/m_z$	Z,Z,Z,Z,Y,Y,Y
14)	2_z	2_{xy}	$4_z2_x2_{xy}$	N,Y,Y,Y,Y,Y,Y
15)	2_z	$m_{\bar{x}y}$	$\bar{4}_z2_xm_{xy}$	Y,Y,Y,Y,Y,Y,Y
16)	2_z	2_{xy}	3_z2_x	N,Y,Y,Y,Y,Y,Y
17)	2_z	m_{xy}	$\bar{3}_zm_x$	Y,Y,Y,Y,Y,Y,Y
18)	2_z	2_1	$6_z2_x2_1$	N,Y,Y,Y,Y,Y,Y
19)	2_z	m_1	$\bar{6}_zm_12_x$	Y,Y,Y,Y,Y,Y,Y
20)*	2_z	$\bar{1}$	$2_z/m_z$	Y,Y,Y,Y,N,N,N
21)	2_z	2_x	$2_z2_y2_x$	N,Y,Y,Y,Y,Y,Y
22)	2_z	m_x	$m_xm_y2_z$	Y,N,Y,Y,Y,Y,Y
23)	2_z	4_z	4_z	N,N,Y,Y,Y,Y,Y
24)	2_z	$\bar{4}_z$	$\bar{4}_z$	Y,Y,Y,Y,Y,Y,Y
25)	2_z	6_z	6_z	N,N,Y,Y,Y,Y,Y
26)	2_z	$\bar{6}_z$	$6_z/m_z$	Y,Y,Y,Y,Y,Y,Y
27)	2_z	3_{xyz}	2_z3_{xyz}	N,Y,Y,Y,Y,Y,Y
28)	2_z	$\bar{3}_{xyz}$	$m_z\bar{3}_{xyz}$	Y,Y,Y,Y,Y,Y,Y
29)	2_{xy}	3_{xyz}	$4_z3_{xyz}2_{xy}$	N,Y,Y,Y,Y,Y,Y
30)	2_{xy}	$\bar{3}_{xyz}$	$m_z\bar{3}_{xyz}m_{xy}$	Y,Y,Y,Y,Y,Y,Y
31)	m_z	$2_{\bar{x}y}$	$\bar{4}_z2_{xy}m_x$	Z,Y,Y,Y,Y,Y,Y
32)	m_z	m_{xy}	3_zm_x	Z,Y,Y,Y,Y,Y,Y
33)	m_z	2_{xy}	$\bar{3}_zm_x$	Z,Y,Y,Y,Y,Y,Y
34)	m_z	m_1	$6_zm_xm_1$	Z,Y,Y,Y,Y,Y,Y
35)	m_z	2_1	$\bar{6}_zm_x2_1$	Z,Y,Y,Y,Y,Y,Y
36)*	m_z	$\bar{1}$	$2_z/m_z$	Z,Y,Y,Y,N,N,N
37)	m_z	2_x	$m_ym_z2_x$	Z,Y,Y,Y,Y,Y,Y
38)	m_z	4_z	$4_z/m_z$	Z,Y,Y,Y,Y,Y,Y
39)	m_z	m_{yz}	$4_zm_ym_{yz}$	Z,Y,Y,Y,Y,Y,Y
40)	m_z	3_z	$\bar{6}_z$	Z,Y,Y,Y,Y,Y,Y
41)	m_z	6_z	$6_z/m_z$	Z,Y,Y,Y,Y,Y,Y
42)	m_z	3_{xyz}	$m_z\bar{3}_{xyz}$	Z,Y,Y,Y,Y,Y,Y
43)	m_{xy}	3_{xyz}	$\bar{4}_z3_{xyz}m_{xy}$	Z,Y,Y,Y,Y,Y,Y
44)	m_{xy}	4_x	$m_z\bar{3}_{xyz}m_{xy}$	Z,Y,Y,Y,Y,Y,Y
45)	$2_z/m_z$	2_{xy}	$\bar{3}_zm_x$	Z,Z,Z,Z,Y,Y,Y
46)	$2_z/m_z$	2_1	$6_z/m_zm_xm_1$	Z,Z,Z,Z,Y,Y,Y
47)	$2_z/m_z$	2_x	$m_xm_ym_z$	Z,Z,Z,Z,Y,Y,Y
48)	$2_z/m_z$	4_z	$4_z/m_z$	Z,Z,Z,Z,Y,Y,Y
49)	$2_z/m_z$	$2_{\bar{x}y}$	$4_z/m_zm_xm_{xy}$	Z,Z,Z,Z,Y,Y,Y
50)	$2_z/m_z$	6_z	$6_z/m_z$	Z,Z,Z,Z,Y,Y,Y
51)	$2_z/m_z$	3_{xyz}	$m_z\bar{3}_{xyz}$	Z,Z,Z,Z,Y,Y,Y
52)	$2_{xy}/m_{xy}$	3_{xyz}	$m_z\bar{3}_{xyz}m_{xy}$	Z,Z,Z,Z,Y,Y,Y
53)*	$2_x2_y2_z$	$\bar{1}$	$m_xm_ym_z$	Y,Z,Y,Y,N,N,N
54)	$2_x2_y2_z$	2_{xy}	$4_z2_x2_{xy}$	N,Z,Y,Y,Y,Y,Y
55)	$2_x2_y2_z$	m_{xy}	$\bar{4}_z2_xm_{xy}$	Y,Z,Y,Y,Y,Y,Y
56)	$2_x2_y2_z$	3_{xyz}	2_z3_{xyz}	N,Z,Y,Y,Y,Y,Y
57)	$2_x2_y2_z$	$\bar{3}_{xyz}$	$m_z\bar{3}_{xyz}$	Y,Z,Y,Y,Y,Y,Y
58)	$2_{xy}2_{\bar{x}y}2_z$	$2_{\bar{x}z}$	$4_z3_{xyz}2_{xy}$	N,Z,Y,Y,Y,Y,Y
59)	$2_{xy}2_{\bar{x}y}2_z$	$m_{\bar{x}z}$	$m_z\bar{3}_{xyz}m_{xy}$	Y,Z,Y,Y,Y,Y,Y
60)	$2_x2_22_z$	2_1	$6_z2_x2_1$	N,Z,Y,Y,Y,Y,Y
61)	$2_x2_22_z$	m_1	$6_z/m_zm_xm_1$	Y,Z,Y,Y,Y,Y,Y
62)*	$m_xm_y2_z$	$\bar{1}$	$m_xm_ym_z$	Z,Y,Y,Y,N,N,N
63)	$m_xm_y2_z$	m_{xy}	$4_zm_xm_{xy}$	Z,N,Y,Y,Y,Y,Y
64)	$m_xm_y2_z$	2_{xy}	$\bar{4}_z2_{xy}m_x$	Z,Y,N,Y,Y,Y,Y
65)	$m_xm_y2_z$	$2_{\bar{y}z}$	$4_z/m_zm_ym_{yz}$	Z,Y,Y,Y,Y,Y,Y
66)	$m_xm_y2_z$	3_{xyz}	$m_z\bar{3}_{xyz}$	Z,Y,Y,Y,Y,Y,Y
67)	$m_{xy}m_{\bar{x}y}2_z$	$m_{\bar{x}z}$	$\bar{4}_z3_{xyz}m_{xy}$	Z,Y,Y,Y,Y,Y,Y

68)	$m_{xy}m_{\bar{x}y}2_z$	$2_{\bar{y}z}$	$m_z\bar{3}_{xyz}m_{xy}$	Z,Y,Y,Y,Y,Y,Y	104)*	3_z	m_z	$\bar{6}_z$	Y,Y,Y,Y,Y,Y,Y
69)	$m_{\bar{x}y}m_z2_{xy}$	$m_{\bar{x}z}$	$m_z\bar{3}_{xyz}m_{xy}$	Z,Y,Y,Y,Y,Y,Y	105)	3_{xyz}	m_x	$m_z\bar{3}_{xyz}$	Y,Y,Y,Y,Y,Y,Y
70)	$m_{\bar{x}y}m_z2_{xy}$	$2_{\bar{y}z}$	$m_z\bar{3}_{xyz}m_{xy}$	Z,Y,Y,Y,Y,Y,Y	106)	3_{xyz}	2_x	2_z3_{xyz}	N,Y,Y,Y,Y,Y,Y
71)	$m_zm_y2_1$	2_2	$\bar{6}_zm_z2_1$	Z,Y,Y,Y,Y,Y,Y	107)	3_{xyz}	m_{yz}	$\bar{4}_z3_{xyz}m_{xy}$	Y,Y,Y,Y,Y,Y,Y
72)	$m_zm_22_z$	2_1	$6_z/m_zm_xm_1$	Z,Y,Y,Y,Y,Y,Y	108)	3_{xyz}	2_{yz}	$4_z3_{xyz}2_{xy}$	N,Y,Y,Y,Y,Y,Y
73)	$m_xm_22_z$	m_1	$6_zm_xm_1$	Z,N,Y,Y,Y,Y,Y	109)*	$\bar{3}_z$	2_z	$6_z/m_z$	Z,Z,Z,Z,Y,Y,Y
74)	$m_xm_22_z$	2_1	$6_z/m_zm_xm_1$	Z,Y,Y,Y,Y,Y,Y	110)*	$\bar{3}_z$	2_x	$\bar{3}_zm_x$	Z,Z,Z,Z,Y,Y,Y
75)	$m_xm_ym_z$	2_{xy}	$4_z/m_zm_xm_{xy}$	Z,Z,Z,Z,Y,Y,Y	111)	$\bar{3}_{xyz}$	2_x	$m_z\bar{3}_{xyz}$	Z,Z,Z,Z,Y,Y,Y
76)	$m_xm_ym_z$	3_{xyz}	$m_z\bar{3}_{xyz}$	Z,Z,Z,Z,Y,Y,Y	112)	$\bar{3}_{xyz}$	2_{yz}	$m_z\bar{3}_{xyz}m_{xy}$	Z,Z,Z,Z,Y,Y,Y
77)	$m_{xy}m_{\bar{x}y}m_z$	$2_{\bar{x}z}$	$m_z\bar{3}_{xyz}m_{xy}$	Z,Z,Z,Z,Y,Y,Y	113)*	3_z2_x	$\bar{1}$	$\bar{3}_zm_x$	Y,Z,Y,Y,N,N,N
78)	$m_xm_2m_z$	2_1	$6_z/m_zm_xm_1$	Z,Z,Z,Z,Y,Y,Y	114)*	3_z2_x	2_z	$6_z2_x2_1$	N,Z,N,Y,Y,Y,Y
79)*	4_z	$\bar{1}$	$4_z/m_z$	Y,Y,Y,Y,N,N,N	115)*	3_z2_x	m_z	$\bar{6}_zm_12_x$	Y,Z,Y,Y,Y,Y,Y
80)*	4_z	2_x	$4_z2_x2_{xy}$	N,Y,N,Y,Y,Y,Y	116)	$3_{xyz}2_{\bar{x}y}$	2_x	$4_z3_{xyz}2_{xy}$	N,Z,Y,Y,Y,Y,Y
81)*	4_z	m_x	$4_zm_xm_{xy}$	Y,N,Y,Y,Y,Y,Y	117)	$3_{xyz}2_{\bar{x}y}$	m_x	$m_z\bar{3}_{xyz}m_{xy}$	Y,Z,Y,Y,Y,Y,Y
82)	4_z	2_{xz}	$4_z3_{xyz}2_{xy}$	N,Y,Y,Y,Y,Y,Y	118)*	3_zm_x	$\bar{1}$	$\bar{3}_zm_x$	Z,Y,Z,Y,N,N,N
83)	4_z	m_{xz}	$m_z\bar{3}_{xyz}m_{xy}$	Y,Y,Y,Y,Y,Y,Y	119)*	3_zm_x	2_z	$6_zm_xm_1$	Z,N,Z,Y,Y,Y,Y
84)*	$\bar{4}_z$	$\bar{1}$	$4_z/m_z$	Z,Z,Y,Y,N,N,N	120)*	3_zm_x	2_1	$\bar{6}_zm_x2_1$	Z,Y,Z,Y,Y,Y,Y
85)*	$\bar{4}_z$	2_x	$\bar{4}_z2_xm_{xy}$	Z,Z,Y,Y,Y,Y,Y	121)	$3_{xyz}m_{\bar{x}y}$	2_x	$\bar{4}_z3_{xyz}m_{xy}$	Z,Y,Z,Y,Y,Y,Y
86)	$\bar{4}_z$	m_{xz}	$\bar{4}_z3_{xyz}m_{xy}$	Z,Z,Y,Y,Y,Y,Y	122)	$3_{xyz}m_{\bar{x}y}$	2_{yz}	$m_z\bar{3}_{xyz}m_{xy}$	Z,Y,Z,Y,Y,Y,Y
87)	$\bar{4}_z$	$2_{\bar{y}z}$	$m_z\bar{3}_{xyz}m_{xy}$	Z,Z,Y,Y,Y,Y,Y	123)	$\bar{3}_{xyz}m_{\bar{x}y}$	2_x	$m_z\bar{3}_{xyz}m_{xy}$	Z,Z,Z,Z,Y,Y,Y
88)*	$4_z/m_z$	2_x	$4_z/m_zm_xm_{xy}$	Z,Z,Z,Z,Y,Y,Y	124)*	$\bar{3}_zm_x$	2_z	$6_z/m_zm_xm_1$	Z,Z,Z,Z,Y,Y,Y
89)	$4_z/m_z$	2_{xz}	$m_z\bar{3}_{xyz}m_{xy}$	Z,Z,Z,Z,Y,Y,Y	125)*	6_z	$\bar{1}$	$6_z/m_z$	Y,Y,Y,Y,N,N,N
90)*	$4_z2_x2_{xy}$	$\bar{1}$	$4_z/m_zm_xm_{xy}$	Y,Z,Y,Y,N,N,N	126)*	6_z	2_x	$6_z2_x2_1$	N,Y,N,Y,Y,N,Y
91)	$4_z2_x2_{xy}$	2_{xz}	$4_z3_{xyz}2_{xy}$	N,Z,Y,Y,Y,Y,Y	127)*	6_z	m_x	$6_zm_xm_1$	Y,N,Y,Y,Y,N,Y
92)	$4_z2_x2_{xy}$	m_{xz}	$m_z\bar{3}_{xyz}m_{xy}$	Y,Z,Y,Y,Y,Y,Y	128)*	$\bar{6}_z$	$\bar{1}$	$6_z/m_z$	Z,Z,Z,Y,N,N,N
93)*	$4_zm_xm_{xy}$	$\bar{1}$	$4_z/m_zm_xm_{xy}$	Z,Y,Z,Y,N,N,N	129)*	$\bar{6}_z$	2_1	$\bar{6}_zm_x2_1$	Z,Z,Z,Y,Y,N,Y
94)	$4_zm_xm_{xy}$	2_{xz}	$m_z\bar{3}_{xyz}m_{xy}$	Z,Y,Z,Y,Y,Y,Y	130)*	$6_z/m_z$	2_x	$6_z/m_zm_xm_1$	Z,Z,Z,Z,Y,N,Y
95)	$\bar{4}_z2_{xy}m_x$	$2_{\bar{x}z}$	$m_z\bar{3}_{xyz}m_{xy}$	Z,Z,Y,Y,Y,Y,Y	131)*	$6_z2_x2_1$	$\bar{1}$	$6_z/m_zm_xm_1$	Y,Z,Y,Y,N,N,N
96)	$\bar{4}_z2_xm_{xy}$	m_{xz}	$\bar{4}_z3_{xyz}m_{xy}$	Z,Z,Y,Y,Y,Y,Y	132)*	$6_zm_xm_1$	$\bar{1}$	$6_z/m_zm_xm_1$	Z,Y,Z,Y,N,N,N
97)	$\bar{4}_z2_xm_{xy}$	$2_{\bar{y}z}$	$m_z\bar{3}_{xyz}m_{xy}$	Z,Z,Y,Y,Y,Y,Y	133)*	$\bar{6}_zm_x2_1$	$\bar{1}$	$6_z/m_zm_xm_1$	Z,Z,Z,Y,N,N,N
98)*	$\bar{4}_z2_xm_{xy}$	$\bar{1}$	$4_z/m_zm_xm_{xy}$	Z,Z,Y,Y,N,N,N	134)*	2_z3_{xyz}	$\bar{1}$	$m_z\bar{3}_{xyz}$	Y,Z,Y,Y,N,N,N
99)	$4_z/m_zm_xm_{xy}$	2_{xz}	$m_z\bar{3}_{xyz}m_{xy}$	Z,Z,Z,Y,Y,Y,Y	135)*	2_z3_{xyz}	2_{xy}	$4_z3_{xyz}2_{xy}$	N,Z,N,Y,Y,N,Y
100)*	3_z	2_x	3_z2_x	N,Y,N,Y,Y,Y,Y	136)*	2_z3_{xyz}	m_{xy}	$\bar{4}_z3_{xyz}m_{xy}$	Y,Z,Y,N,Y,N,Y
101)*	3_z	m_x	3_zm_x	Y,N,Y,Y,Y,Y,Y	137)*	$\bar{4}_z3_{xyz}m_{xy}$	$\bar{1}$	$m_z\bar{3}_{xyz}m_{xy}$	Z,Z,Z,Y,Z,N,N
102)*	3_z	$\bar{1}$	$\bar{3}_z$	Y,Y,Y,Y,N,N,N	138)*	$4_z3_{xyz}2_{xy}$	$\bar{1}$	$m_z\bar{3}_{xyz}m_{xy}$	Y,Z,Y,Z,Z,N,N
103)*	3_z	2_z	6_z	N,N,N,Y,Y,Y,Y	139)*	$m_z\bar{3}_{xyz}$	2_{xy}	$m_z\bar{3}_{xyz}m_{xy}$	Z,Z,Z,Z,Y,N,Y

References

[1] V. Janovec, D.B. Litvin & L. Richterová, *Ferroelectrics,* **157**, 75 (1994).
[2] K. Aizu, *Phys. Rev.,* **B2,** 754 (1970).
[3] D.B. Litvin, *Acta Cryst.,* **A40,** 255 (1984).
[4] D.B. Litvin, *Acta Cryst.,* **A41,** 89 (1985).
[5] V. Janovec, L. Richterová & D.B. Litvin, *Ferroelectrics,* **126,** 287 (1992).
[6] V. Janovec, L. Richterová & D.B. Litvin, *Ferroelectrics,* **140,** 95 (1993).
[7] D.B. Litvin & V. Janovec, *Ferroelectrics,* **191,** 9 (1997).
[8] J.F. Nye, *Physical Properties of Crystals* (Clarendon Press, Oxford, 1957).
[9] V. Janovec, *Czech. J. Phys.,* **B22,** 974 (1972).
[10] D.B. Litvin & T.R. Wike, *Ferroelectrics,* **98,** 157, (1989).
[11] J. Fuksa & V. Janovec, *Ferroelectrics,* these proceedings.
[12] V. Janovec, D.B. Litvin & J. Fuksa, *Ferroelectrics,* **172,** 351 (1995).